MOW, MBW pin measurement formulas [in.]

Basic formulas:

$$D_p = \frac{Z}{d_0}$$

$$d_b = d_0 * \cos\phi \ [in.]$$

$$t = \frac{\pi}{2D_p} \ [in.]$$

Conversion from degrees to radians

$$\hat{\phi} = \frac{\pi\phi}{180} \ [rad.]$$

Standard measuring pin diameter [in.]

$$d_w = \frac{1.728}{D_p}$$

Where:
$Z - number\ of\ teeth$
$d_0 -\ pitch\ diameter\ [in.]$
$d_b - base\ diameter\ [in.]$
$D_p - diametral\ pitch\ [dimensionless]$
$\phi - pressure\ angle\ [degree]$
$t - CTT, circular\ tooth\ thickness\ [in.]$

General form of an involute function

$\hat{\Theta} = inv\phi = \tan\phi - \hat{\phi}$

Where:
$\hat{\Theta} - polar\ angle\ [radians]$
$\Theta - polar\ angle\ [degree]$
$inv\phi -\ involute\ function\ of\ an\ angle\ [rad.]$
$\phi - involute\ pressure\ angle\ [degree]$
$\hat{\phi} - involute\ pressure\ angle\ [radians]$

Involute function at the center of measuring pin [radians]

$$\hat{\Theta}_{cw} = inv\phi_{cw} = \left(\frac{t}{d_0}\right) + \left(\frac{d_w}{d_b}\right) + inv\phi - \frac{\pi}{Z}$$

Where:
$\phi_{cw} -\ pressure\ angle\ at\ the\ center\ of\ measuring\ pin\ [degree]$
$inv\phi_{cw} -\ involute\ function\ at\ \phi_{cw}\ [radians]$
$\hat{\Theta}_{cw} -\ polar\ angle\ at\ \phi_{cw}\ [radians]$
$\Theta_{cw} -\ polar\ angle\ at\ \phi_{cw}\ [degree]$
$d_0 -\ pitch\ diameter\ [in.]$
$d_w - measuring\ pin\ diameter\ [in.]$
$d_b - base\ diameter\ [in.]$
$inv\phi - involute\ function\ of\ the\ pressure\ angle\ [radians]$
$Z - number\ of\ teeth$

Pressure angle at the center of measuring pin [degrees]

$$\phi_{cw} = arcinv\phi_{cw}$$

Where:

$arcinv\phi_{cw} - reverse\ involute\ function$

The value of ϕ_{cw}, in degrees, can be found in tables (reference "Involutometry and Trigonometry" by Werner F. Vogel) or may be calculated utilizing Excel Solver, TkSolver or scientific calculator. The solution can also be found graphically.

MOW (external gear, even number of teeth) [in.]

$$MOW_{even} = \frac{d_b}{cos\phi_{cw}} + d_w$$

MOW (external gear, odd number of teeth) [in.]

$$MOW_{odd} = \frac{\left(d_b * cos\ \frac{90}{Z}\right)}{cos\phi_{cw}} + d_w$$

MBW (Internal gear, even number of teeth) [in.]

Involute function at the center of measuring wire [radians]

$$\hat{\Theta}_{cw} = inv\phi_{cw} = \left(\frac{s}{d_0}\right) + inv\phi - \left(\frac{d_w}{d_b}\right)$$

Where:

$s - CSW, circular\ space\ width\ [in.]$

$$MBW_{even} = \frac{d_b}{cos\phi_{cw}} - d_w$$

MBW (Internal gear, odd number of teeth) [in.]

$$MBW_{odd} = \frac{\left(d_b * cos\ \frac{90}{Z}\right)}{cos\phi_{cw}} - d_w$$

Example:

External gear
$Z = 20\ teeth$
$D_p = 20$
$\phi = 20\ [deg.]$

$$d_0 = \frac{Z}{D_p} = \frac{20}{20} = 1.0000\ [in.]$$

$$d_b = d_0 * \cos\phi = 1.0000 * \cos 20^0 = 0.9397\ [in.]$$

$$d_w = \frac{1.728}{D_p} = \frac{1.728}{20} = 0.0864\ [in.]$$

$$t = \frac{\pi}{2D_p} = \frac{\pi}{2 * 20} = 0.0785\ [in.]$$

$$\hat{\phi} = \frac{\pi\phi}{180} = \frac{\pi * 20}{180} = 0.3491\ [rad.]$$

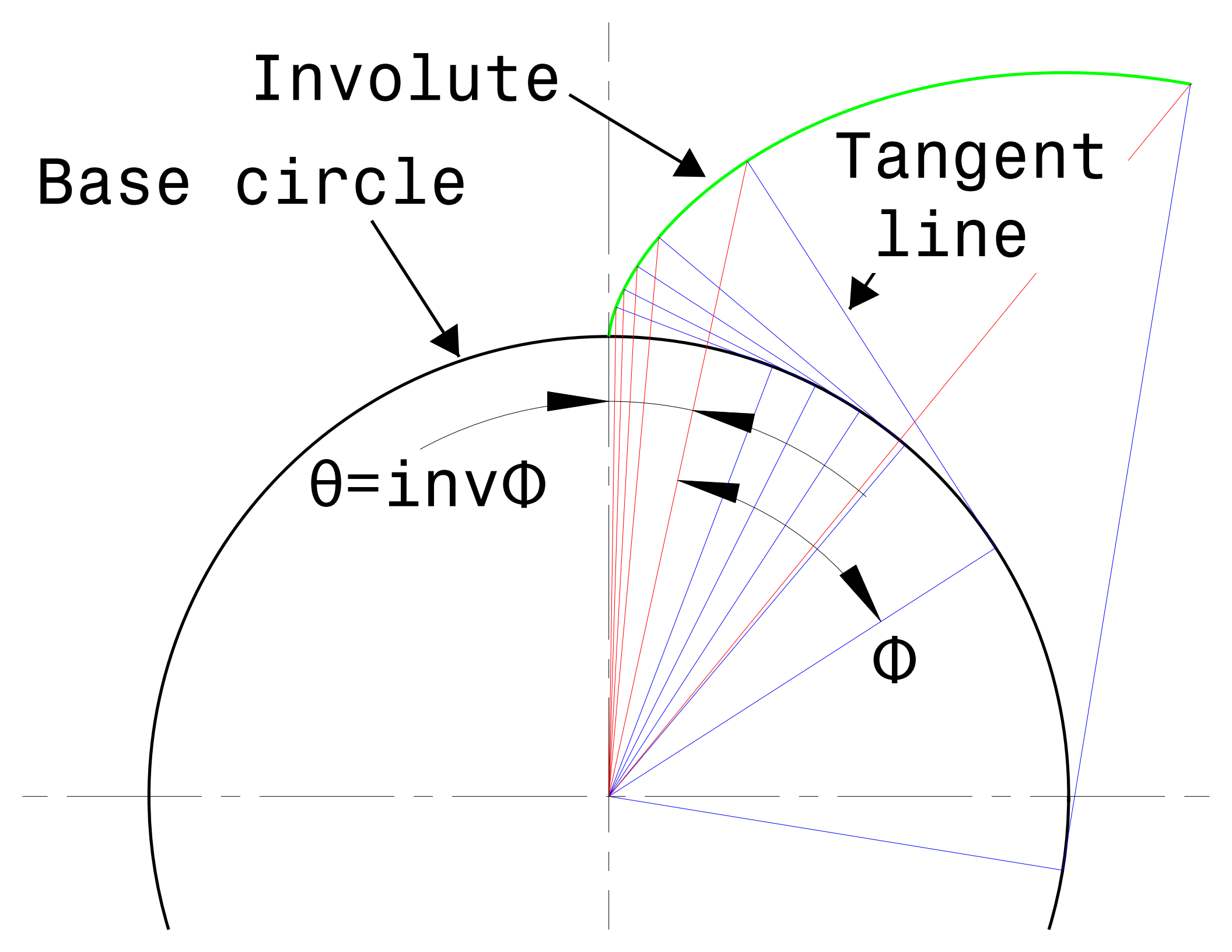
$$inv\phi = \tan\phi - \hat{\phi} = \tan 20 - 0.3491 = 0.0149\ [rad.]$$

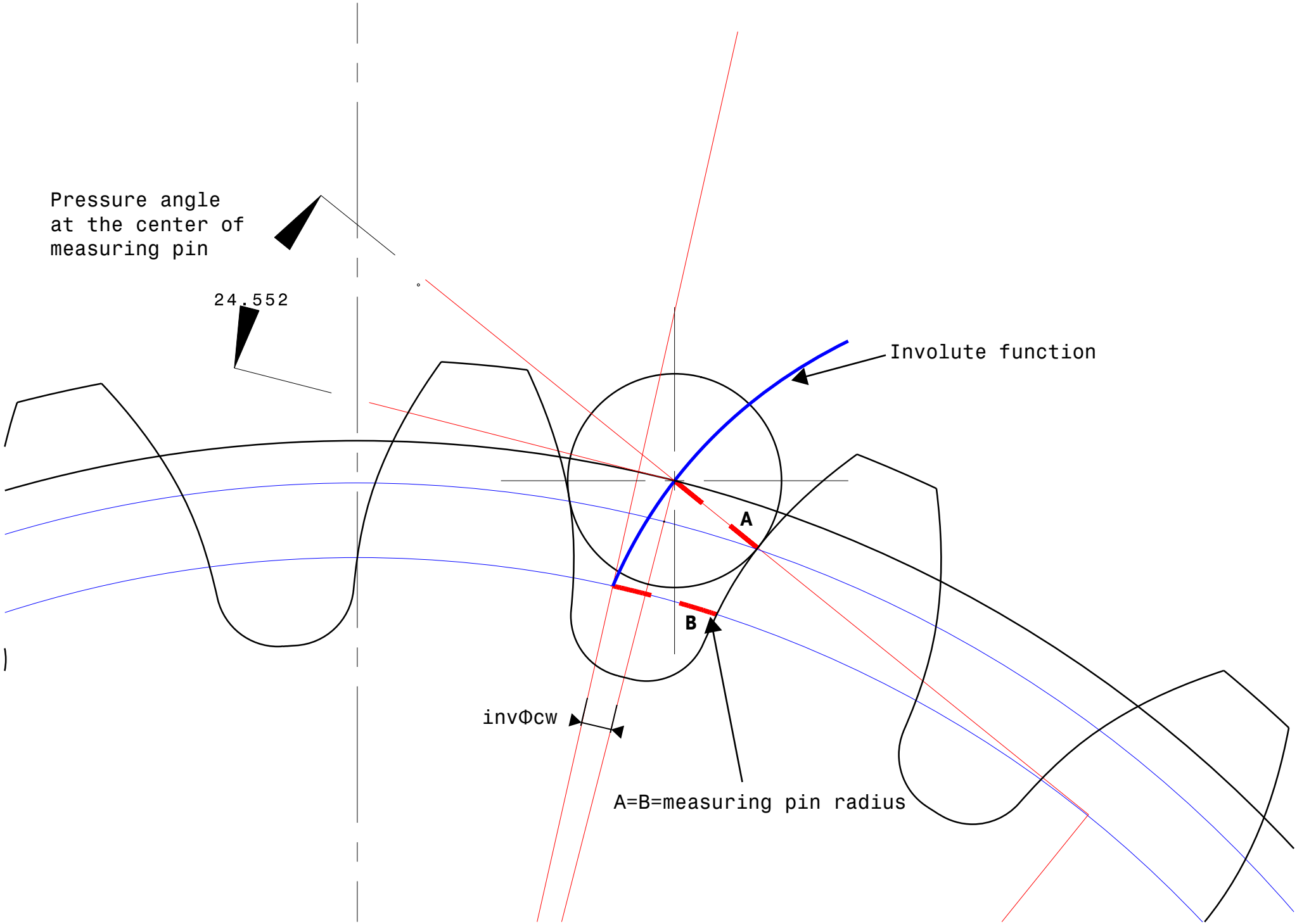
$$\hat{\Theta}_{cw} = inv\phi_{cw} = \left(\frac{t}{d_0}\right) + \left(\frac{d_w}{d_b}\right) + inv\phi - \frac{\pi}{Z} = \left(\frac{0.0785}{1.0000}\right) + \left(\frac{0.0864}{0.9397}\right) + 0.0149 - \frac{\pi}{20} = 0.0283\ [rad.]$$

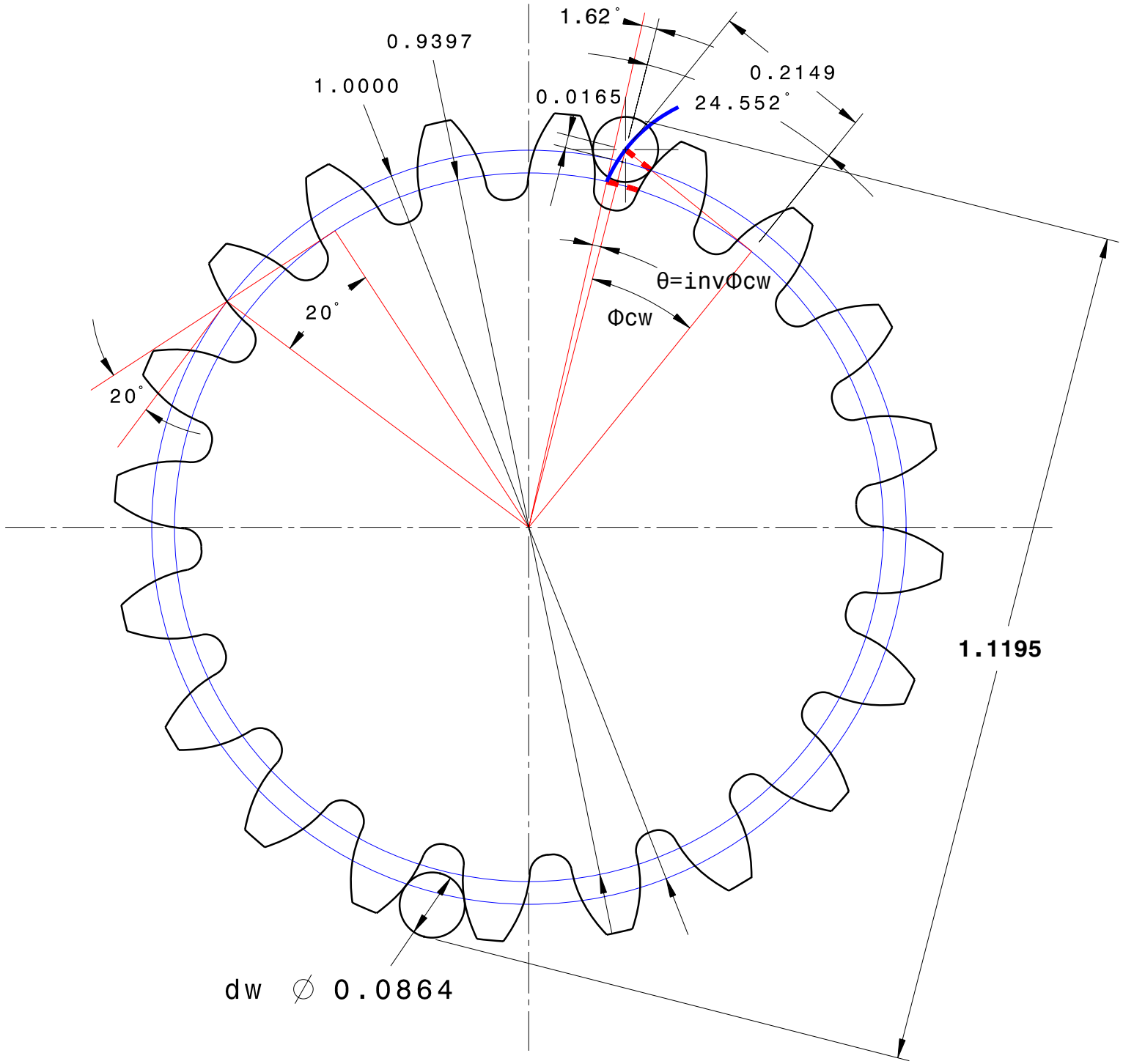
$$\Theta_{cw} = \frac{180 * \hat{\Theta}_{cw}}{\pi} = \frac{180 * 0.0283}{\pi} = 1.62°$$

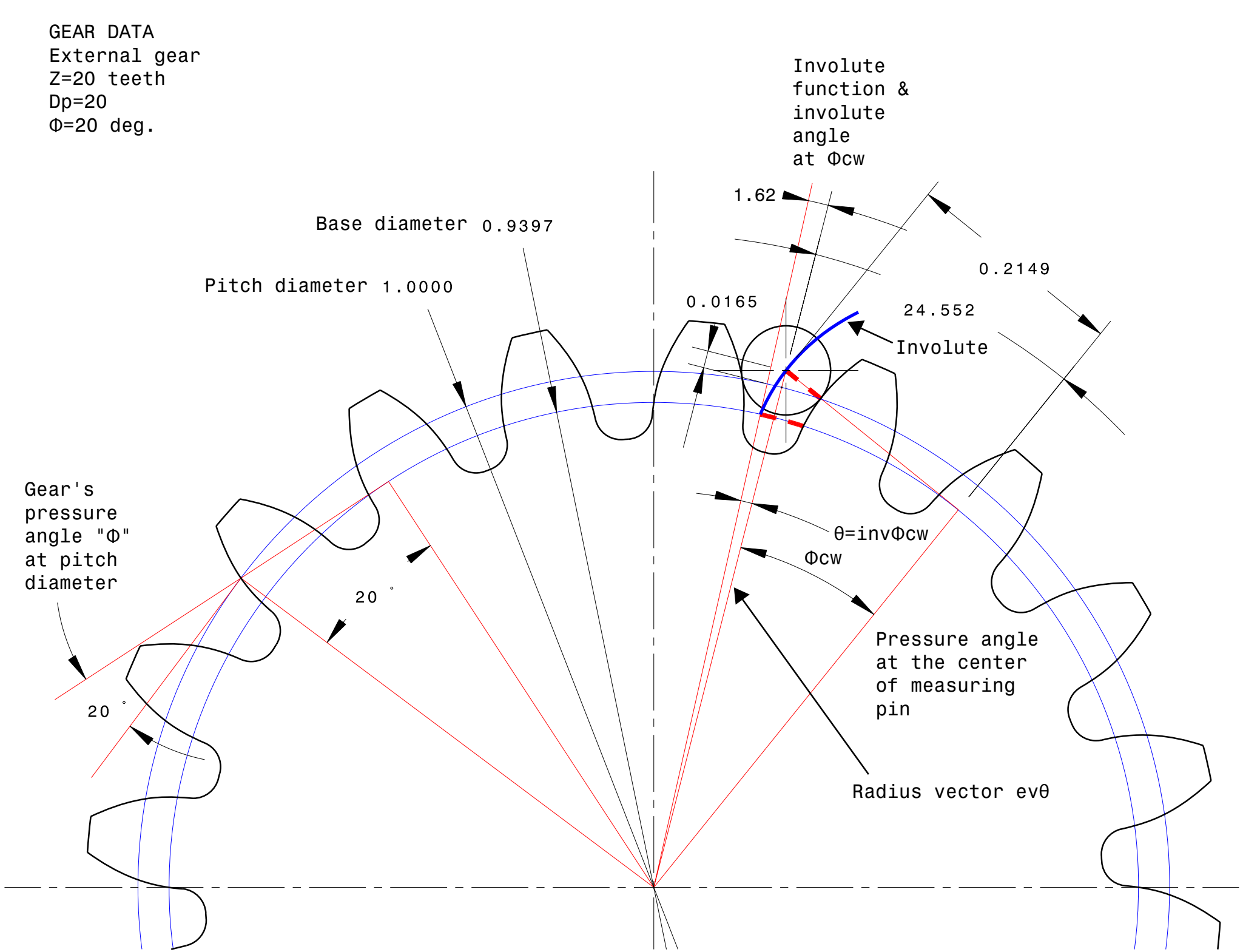
From Excel Solver:
$\phi_{cw} = arcinv\phi_{cw} = 24.55\ [deg.]$

$$MOW_{even} = \frac{d_b}{cos\phi_{cw}} + d_w = \frac{0.9397}{\cos\ 24.55°} + 0.0864 = 1.1195\ [in.]$$


Involute
Base circle
Tangent line
θ=invΦ
Φ


Pressure angle
at the center of
measuring pin
24.552
°
Involute function
A
B
invΦcw
A=B=measuring pin radius


1.62°
0.9397
1.0000
0.0165
24.552°
0.2149
θ=invΦcw
Φcw
20°
20°
1.1195
dw ∅ 0.0864


GEAR DATA
External gear
Z=20 teeth
Dp=20
Φ=20 deg.
Involute function & involute angle at Φcw
1.62
Base diameter 0.9397
Pitch diameter 1.0000
0.0165
0.2149
24.552
Involute
Gear's pressure angle "Φ" at pitch diameter
20 °
20 °
θ=invΦcw
Φcw
Pressure angle at the center of measuring pin
Radius vector evθ